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ABSTRACT

This article is a reflection on the mathematical legacy of Professor Petr Simon.

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1. Introduction

The prominent Czech topologist Prof. Petr Simon passed away on April 14th, 2018. He is an important link in the chain of renowned Czech topologists which includes well-known names like those of Eduard Čech, Miroslav Katětov, and Zdeněk Frolík. In this note we wish to review his many achievements, where we concentrate on his scientific contributions to the field of Set-Theoretic Topology.

Petr Simon was born on the 24th of February 1944 in Hradec Králové in what is now the Czech Republic. He attended elementary school and secondary school in Prague, and studied at the Faculty of Mathematics and Physics of the Charles University in Prague in the period 1961–1966. Upon completing his studies he joined the Faculty as a Research Assistant in 1967; he worked there (mostly as a researcher) for the rest of his life. In 1977 he successfully defended his CSc (Candidate of Science, roughly equivalent to a PhD) dissertation *O lokálním merotopickém characteru* (On local merotopic character) under the supervision of Professor Miroslav Katětov; this came with an increase in rank to *Researcher in mathematics*. He was promoted to the rank of *Independent researcher* in 1986 and, after defending his DrSc habilitation *Aplikace ultrafiltrů v topologii* (Applications of ultrafilters in topology), to *Leading researcher* in 1990. Finally, in 2001 he was awarded the title of full professor at Charles University.

When Petr Simon entered the mathematical life at the Charles University there was a very active group of researchers in Topology, Set Theory and related areas. There was the *Topological seminar* led by Miroslav Katětov, the *Set Theory seminar* of Petr Vopěnka, Věra Trnková's *Category Theory seminar*, and the seminars on *Measure theory* and *Uniform spaces* organized by Zdeněk Frolík. These were attended over the years by a talented group of young mathematicians including Jiří Adámek, Bohuslav Balcar, Lev Bukovský, Jan Hejcman, Petr Holický, Karel Hrbáček, Miroslav Hušek, Václav Koubek, Luděk Kučera, Věra Kůrková, Vladimír Müller, Jaroslav Nešetřil, Tomáš Jech, Jan Pachl, Jan Pelant, David Preiss, Karel Příkrý, Jan Reiterman, Vojtěch Rödl, Jiří Vilímovský, Jiří Vinárek, and Miloš Zahradník, to mention but a few.

Every five years since 1961, Prague hosts the prestigious *Topological Symposium* an initiative of Eduard Čech. Petr Simon has participated in all but the first two of them, first as a speaker, and since 1981 as an organizer, and in 2006 as its chairman. He also participated actively in the organization of the *Winter School in Abstract Analysis, Section Topology*. These schools have been organized continuously since 1973 and were originally created by Zdeněk Frolík as a winter getaway for the members of his seminars, but they quickly grew into important events in several fields (including, besides real and functional analysis, set theory and topology, also category theory and combinatorics). See [73] for a short history.

Petr Simon was a longtime member of the editorial board of *Topology and Its Applications* (from 1992 till 2018) and the managing editor of *Acta Universitatis Carolinae Mathematica et Physica* (1989–2010). He was the representative of Czech Set Theory in the European Set Theory Society and the INFTY project (2009–2014).

To the best of our knowledge, Petr Simon was the advisor of the following doctoral students:

- Egbert Thümmel (1996): *Ramsey theorems and topological dynamics*
- Eva Murtinová (2002): *Separation Axioms in dense subsets*
- Jana Flašková (2006): *Ultrafilters and small sets*
- David Chodounský (2011): *On the Katowice problem*
- Jonathan Verner (2011): *Ultrafilters and Independent systems*
- Jan Starý (2014): *Complete Boolean Algebras and Extremally Disconnected Compact Spaces*

He has also supervised the Master's thesis of Dana Bartošová, and advised (albeit briefly) as PhD students Michael Hrušák and Adam Bartoš.

Petr Simon has published more than 70 research articles and participated in the preparation of the *Handbook of Boolean Algebras*. He co-edited the books *The Mathematical Legacy of Eduard Čech* and *Recent Progress in General Topology III*.

His main contributions to topology lie in a thorough study of the combinatorial structure of the *space of ultrafilters* $\beta\mathbb{N}$ via almost disjoint and independent families, as well as the corresponding cardinal invariants of the continuum. His most lethal weapons were maximal almost disjoint (MAD) families and ultrafilters (*the most beautiful things there are* — he used to say).

He is and shall be missed by us and the whole community of Set Theory and Set-theoretic Topology. In what follows we take a more detailed look at some of his contributions to mathematics.

Note to the reader: we partitioned the references into two families. The first, cited as [Sm], consists of works (co-)authored by Petr Simon; the second, cited as [On], contains material related to Simon's work.

2. Early beginnings

Petr Simon published his first two articles in 1971 in the same volume of *Commentationes Mathematicae Universitatis Carolinae* (CMUC), fittingly, the journal founded in 1960 by his academic grandfather Eduard Čech.[1]

In the first paper [1] he made important contributions to the study of *merotopic spaces*, introduced by Miroslav Katětov in [109], by John Isbell under the name *quasi-uniform spaces* in [107] and again by Horst Herrlich in [106] as *quasinearness spaces* — a topic he revisited in [9] and which formed part of his dissertation. He would later return to general continuity structures in his study of atoms in the lattice of uniformities and their relation to ultrafilters [7,34,35], the latter two papers form joint work with Jan Pelant, Jan Reiterman and Vojtěch Rödl, research undoubtedly stimulated by Zdeněk Frolík's research seminar on Uniform Spaces.

In the second paper, [2] (see also [3]), he studied topological properties of Mary Ellen Rudin's Dowker space, the then recently published first ZFC example of such a space [112].

In [4] he contributed to the, then recently initiated, study of cardinal functions on topological spaces by proving that the cellularity of the square of a linearly ordered topological space equals the density of the space itself. This contains Kurepa's result from [111] that for a linearly ordered space the cellularity of the square is not larger than the successor of the cellularity of the space itself.

A joint paper with David Preiss [5] contains a proof of the fact that a pseudocompact subspace of a Banach space equipped with the weak topology is compact. A key component of the proof extracts an important property of *Eberlein compacta*[2] later dubbed the Preiss-Simon property: a space X has the *Preiss-Simon property* if for every closed $F \subseteq X$, each point $x \in F$ is a limit of a sequence of non-empty open subsets of F. He continued the study of Eberlein compact spaces in [6], providing partial results toward proving that continuous images of Eberlein compacta are Eberlein; this was proved soon thereafter by Yoav Benyamini, Mary Ellen Rudin and Michael Wage in [98].

One of Petr Simon's strengths was his ability to construct ingenious examples and counterexamples in topology. Answering a question raised by J. Pelham Thomas in [119] he constructed in [10] the first example of an infinite maximal connected Hausdorff space, that is, a connected Hausdorff topological space X such that any finer topology on X is disconnected. In [12,15] he constructed a compactification $b\mathbb{N}$ of the countable discrete space $\mathbb{N}$ with a sequentially compact remainder such that no sequence in $\mathbb{N}$ converges in $b\mathbb{N}$. For later examples, one can look at his construction in [79] of a connected metric space in which every infinite separable subspace is not connected, or a joint work with Steve Watson [51] where a completely regular

[1] The first volume of CMUC contains only two papers. One by Eduard Čech and the other by Miroslav Katětov.
[2] A topological space is Eberlein compact if it is homeomorphic to a weakly compact subset of a Banach space.

space which is connected, locally connected and countable dense homogeneous, but not strongly locally homogeneous is constructed.

3. Almost disjoint families and ultrafilters

The *Novák number* $\mathfrak{n}(X)$[3] of a topological space without isolated points is the minimum cardinality of a family of nowhere dense subsets that covers it. Petr Simon first used the Novák number in [13,14] to give alternative proofs of a result of Mikhail Tkachenko.

The study of Novák number was a popular research topic in Prague. For example, Petr Štěpánek and Petr Vopěnka showed in [118] that the Novák number of any nowhere separable metric space is at most $\aleph_1$. Petr Simon [8] greatly generalized their result, and then together with Bohuslav Balcar and Jan Pelant he studied the behaviour of the Novák number of the Čech-Stone remainder $\mathbb{N}^* = \beta\mathbb{N} \setminus \mathbb{N}$ of $\mathbb{N}$ in [17]. In this important paper, they showed that the value of $\mathfrak{n}(\mathbb{N}^*)$ depends heavily on set-theoretic axioms. To study these they introduced the *distributivity number* $\mathfrak{h}$ of the Boolean algebra $\mathcal{P}(\mathbb{N})/\mathit{fin}$ and proved their celebrated and influential *Base Tree Theorem* which states that the algebra has a dense subset T which is a $\mathfrak{c}$-branching tree of height $\mathfrak{h}$, and showed that (1) if $\mathfrak{h} < \mathfrak{c}$ then $\mathfrak{h} \leq \mathfrak{n}(\mathbb{N}^*) \leq \mathfrak{h}^+$, (2) if $\mathfrak{h} = \mathfrak{c}$ then $\mathfrak{h} \leq \mathfrak{n}(\mathbb{N}^*) \leq 2^{\mathfrak{c}}$, and (3) $\mathfrak{h} = \mathfrak{n}(\mathbb{N}^*)$ if and only if there is a Base Tree without cofinal branches.

Simon quickly put the theorem to further use in [11] to show that if $\mathfrak{n}(\mathbb{N}^*) > \mathfrak{c}$ then $\mathbb{N}^*$ contains a dense linearly ordered subspace, in [61] where it is shown that $\mathfrak{h}$ is the minimal size of a family of sequentially compact spaces whose product is not sequentially compact and in [68] where he constructs a σ-centered atomic, almost rigid tree-like Boolean algebra such that every injective endomorphism is onto, and every surjective homomorphism is injective.

The paper [8] was also the start of a long and fruitful collaboration of Petr Simon with Bohuslav Balcar. The two have co-authored over twenty publications and co-directed the well-known *Seminář z počtů*[4] (loosely translated as “Seminar on reckoning” or “Seminar on counting”) and together raised a new generation of Czech Set-Theorists and Topologists.

Together with Peter Vojtáš [20,19] (see also [26,39]) they continued the project initiated by Balcar and Vojtáš in determining which subsets of Boolean algebras admit a disjoint refinement. To discuss a particularly important case of the above mentioned phenomenon let us introduce some notation first: A family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ is *almost disjoint* (AD) if every two distinct elements of $\mathcal{A}$ have finite intersection. An infinite AD family is *maximal* (MAD) if it is maximal with respect to inclusion or, equivalently, if for every infinite $B \subseteq \mathbb{N}$ there is an $A \in \mathcal{A}$ such that $B \cap A$ is infinite. Given a MAD family $\mathcal{A}$ we denote by $\mathcal{I}(\mathcal{A})$ the family of $\mathcal{A}$-small subsets of $\mathbb{N}$: those sets which have infinite intersection with only finitely many elements of $\mathcal{A}$, and by $\mathcal{I}^+(\mathcal{A})$ the family of those subsets of $\mathbb{N}$ which are not $\mathcal{A}$-small, and therefore are called $\mathcal{A}$-*large*.

The crucial problem, first studied in [26], is RPC(ω):

> Given a maximal almost disjoint family $\mathcal{A}$, does there exist an almost disjoint refinement for the family of all $\mathcal{A}$-large sets?

A closely related problem was formulated independently and in a different context by Paul Erdős and Saharon Shelah in [102]:

> Is there a completely separable MAD family, that is, is there a MAD family $\mathcal{A}$ that is itself an almost disjoint refinement for the family of $\mathcal{A}$-large sets?

[3] The *Baire number* for the rest of the world, and eventually even for the Czech community.

[4] The name is a play on the curious fact that, at the time, set theory was being introduced in mathematics classes taught in lower elementary school, these classes were then called “reckoning”.

The problem RPC(ω) has the following two topological equivalents

(1) for every nowhere dense subset N of $\mathbb{N}^*$ there is a family of $\mathfrak{c}$-many disjoint open subsets of $\mathbb{N}^*$ each having N in its closure, and
(2) for every nowhere dense subset N of $\mathbb{N}^*$ there is a nowhere dense subset M of $\mathbb{N}^*$ such that N is a nowhere dense subset of M, see [45].

It is believed that both problems have positive solutions in ZFC, and Simon and collaborators have made steady progress toward a solution [26,39,69]. Currently the best result is due to Saharon Shelah who showed in [115] that the answer is positive if $\mathfrak{c} < \aleph_\omega$ and that a negative solution would imply consistency of the existence of large cardinals.

Having completed their study of the structure of the Boolean algebra $\mathcal{P}(\omega)/\mathit{fin}$, Simon and Balcar turned to the higher cardinal analogues $\mathcal{P}(\kappa)/[\kappa]^{<\kappa}$, a study initiated by Balcar and Vopěnka in [97] and the corresponding study of *uniform ultrafilters on* κ. In [37,39], extending earlier results of Balcar and Vopěnka, they showed that $\mathfrak{h}_\kappa = \aleph_0$ for cardinals κ of uncountable cofinality and $\mathfrak{h}_\kappa = \aleph_1$ for uncountable cardinals κ of countable cofinality,[5] while in [63] they computed the Novák numbers of the space $U(\kappa)$ of *uniform ultrafilters on* κ, i.e., the Stone space of the Boolean algebra $\mathcal{P}(\kappa)/[\kappa]^{<\kappa}$. They proved that (1) $\mathfrak{n}(U(\kappa)) = \aleph_1$ for all κ of uncountable cofinality, and (2) $\mathfrak{n}(U(\kappa)) = \aleph_2$ for all κ of countable cofinality, assuming one of $\neg$CH, $2^{\aleph_1} = \aleph_2$, and $\kappa^{\aleph_0} = 2^\kappa$ holds. They further studied which cardinals are being collapsed in generic extensions by $\mathcal{P}(\kappa)/[\kappa]^{<\kappa}$ in [37,39,78]; the final word on this is yet to be written. Currently the best partial results, by Saharon Shelah, appear in [116].

Continuing the work of Balcar and Vojtáš, they further studied the structure of $U(\kappa)$ along the lines of RPC, focusing on a question of Wis Comfort and Neil Hindman from [99]: *Given an arbitrary cardinal κ, is each point in $U(\kappa)$ a κ^+-point*, i.e., is there a family of κ^+ pairwise disjoint open subsets of $U(\kappa)$ each containing the point in its closure? They showed in [23] that the answer is positive for regular cardinals and later Simon showed in [25] that it is also positive for cardinals of countable cofinality. For such cardinals κ, answering another question of Wis Comfort, he showed in [83] that there exist uniform ultrafilters on κ that cannot be obtained from the set of all sub-uniform ultrafilters by iterating the closure of sets of size less than κ.

In a pair of articles [41,52] Balcar and Simon investigated the minimal π-character of ultrafilters on Boolean algebras. They showed that if a Boolean algebra is homogeneous or complete then the minimal π-character coincides with the *reaping number* of the Boolean algebra, which is defined as the minimal size of a family of non-zero elements such that no element splits them all. Then they used their results to show that every extremely disconnected ccc space in which π-weight and minimal π-character coincide, contains a point which is not an accumulation point of a countable discrete set, also known as a *discretely untouchable point* (see [48]).

To put this result in context, we recall that Zdeněk Frolík proved in [105] that every infinite compact extremely disconnected space (the Stone space of a complete Boolean algebra) is not homogeneous. This proof, however, did not produce examples of simple topological properties shared by some but not all of the points of the space. It is clear that every compact space contains points which are accumulation points of countable discrete sets, so “being discretely untouchable” is a property not shared by all points in a compact space — if it were shown to be shared by some then one would have a more concrete reason for non-homogeneity.

The non-homogeneity of $\beta\mathbb{N}\setminus\mathbb{N}$ was proved under CH by Walter Rudin in [113] using the simple property of being a *P-point* (the intersection of countably many neighbourhoods is again a neighbourhood), in ZFC by Zdeněk Frolík in [104] using a rather unwieldy invariant of points in $\beta\mathbb{N}\setminus\mathbb{N}$ that involved copies of the

[5] Here we take as definition of $\mathfrak{h}_\kappa$ the minimal μ such that forcing with $\mathcal{P}(\kappa)/[\kappa]^{<\kappa}$ adds a new set of size μ.

space inside itself, and again in ZFC by Ken Kunen in [110] using the property of being a *weak P-point* (not an accumulation point of any countable set). This latter proof introduced a powerful method of constructing ultrafilters using independent linked families.[6] This method was used by Simon in [27] and [30] to construct ultrafilters on $\mathbb{N}$ that are minimal in the Rudin-Frolík order, and in [28,32] to give a ZFC construction of a closed separable subspace of $\mathbb{N}^*$ that is not a retract of $\beta\mathbb{N}$.

That last construction is highly involved and takes about ten pages. It also illustrates what some us know well, namely that Petr Simon had a way with words; after an outline of the construction and just before the hard work starts we find the following gem: "so let us begin now to swallow the indigestible technicalities".

In joint work Petr Simon, Murray Bell and Leonid Shapiro [65] showed that a large class of spaces, called *orthogonal* $\mathbb{N}^*$*-images*, that includes all separable compact spaces, all compact spaces of weight at most $\aleph_1$ and all perfectly normal compact spaces, is closed under products of $\mathfrak{c}$-sized families. A compact space K is an orthogonal $\mathbb{N}^*$-image if $\mathbb{N}^* \times K$ is a continuous image of $\mathbb{N}^*$. This result — trivially true under CH by Parovichenko's Theorem — puts some limits to the *rigidity phenomena* that one encounters assuming PFA, see, e.g., Farah [103] and Dow-Hart [100].

The Stone duality that links zero-dimensional compact Hausdorff spaces and Boolean algebras permeates much of Petr Simon's work. In a joint paper with Martin Weese [29] he constructed non-homeomorphic *thin-tall* scattered spaces, later superseded by his work with Alan Dow in [50]. In [80], answering a question of Alexander Arkhangel'skiĭ, he showed that there is a separable compact Hausdorff space X having a countable dense-in-itself, dense set D which is a P-set in X.

Another constant in Simon's work is the use of cardinal invariants of the continuum, both as tools [38,44] and as principal objects of study [56,85,92,95]. For example, the first of these four paper shows that Sacks forcing collapses $\mathfrak{c}$ to $\mathfrak{b}$.

In a joint paper with Alan Dow and Jerry Vaughan [38] he found one of the first applications of Set Theory in Algebraic Topology and Homological Algebra. The study of the interactions between these fields has started to flourish only recently.

4. Convergence properties

Petr Simon made a strong impact on the study of Fréchet spaces and their generalizations, often in collaboration with his Italian colleagues Angelo Bella, Camilo Costantini and Gino Tironi. Recall that a topological space X is *Fréchet* if for every point in the closure of a set $A \subseteq X$ there is a sequence of elements of A that converges toconverging to it. There is an extremely close connection between Fréchet spaces and almost disjoint families.

In [16] Simon gave the first ZFC example of a compact Fréchet space whose square is not Fréchet. The result followed using known techniques from the fact that there is a MAD family $\mathcal{A}$ that can be partitioned into two subfamilies, each of which is not maximal when restricted to an $\mathcal{A}$-large set, they are thus called *nowhere maximal*. Simon's proof of this is a thing of beauty, a *proof from the book*, a short concise proof by contradiction. He later also used this result in [24].

In [75] and [81] Simon studied the question of Tsugunori Nogura, whether the product of two Fréchet spaces neither of which contains containing a copy of the sequential fan can contain a copy of it. First, [75] he showed that assuming CH such spaces can be constructed, and later in joint work with Tironi, [81], gave partial results in the opposite direction. The final solution to the problem was given by Stevo Todorčević in [120] extending the approach of Simon and Tironi: the Open Colouring Axiom implies a positive answer.

On the other hand, Costantini and Simon [76] answering another question of Nogura gave a ZFC example of two Fréchet spaces, the product of which does not contain a copy of the sequential fan but fails to be

[6] The example of such a family in Kunen's paper is given by an explicit formula due to Petr Simon, rather than Kunen's own construction "involving trees of trees".

Fréchet. For this they used a construction of an AD family with properties resembling those of a completely separable MAD family. Later Bella, Costantini and Simon [88] showed that assuming CH one can construct Fréchet spaces containing a copy of the sequential fan which are pseudocompact. In his last paper concerning the subject [94] Simon further advanced these techniques to construct a Fréchet space not containing a copy of the sequential fan all of whose finite powers are Fréchet.

By allowing transfinite sequences when reaching points in the closure one arrives at a more general notion of a *radial space* or a *Fréchet-chain-net space*. If one only requires that every non-closed set contains a transfinite sequence converging outside of the set, one gets the notion of a *pseudoradial space* or a *chain-net space*, just as with the usual convergence one defines a *sequential space*. Simon started to study such spaces in a joint paper [22] with Ignacio Jané, Paul Meyer and Richard Wilson where they, assuming CH, constructed Hausdorff examples of countably tight pseudoradial spaces which are not sequential. In a paper with Tironi [31] they produced a ZFC example. Completely regular examples were given soon after by Juhász and Weiss in [108]. In [64,74] and [77] Simon and co-authors look at products of (compact) pseudo-radial spaces.

A further weakening gives rise to Whyburn and weakly Whyburn spaces: We say X is a *Whyburn* space if whenever $x \in \overline{A} \setminus A$, there is a $B \subseteq A$ such that $\overline{B} \setminus A = \{x\}$, and X is *weakly Whyburn* if whenever $A \subseteq X$ is not closed, there is a $B \subseteq A$ such that $|\overline{B} \setminus A| = 1$. Simon first studied these spaces in [62] (using different terminology) and showed that there are two Whyburn spaces whose product is not weakly Whyburn. In [90], assuming CH, Bella and Simon construct a pseudocompact Whyburn space of countable tightness that is not Fréchet.

The paper [82] of Bella and Simon continues the study initiated in [101] by Dow, Tkachenko, Tkachuk and Wilson of *discretely generated* spaces: spaces where points in the closure can be reached by discrete sets — another weakening of radiality. They show that countably tight countably compact spaces are discretely generated and show that this consistently fails for pseudo-compact spaces of countable tightness.

The paper [82] also contains results concerning spaces of continuous functions endowed with the topology of pointwise convergence. In particular, it shows that if X is σ-compact then $C_p(X)$ is discretely generated. The work with Tsaban [92] shows that the *pseudo-intersection number* $\mathfrak{p}$ is the minimal cardinality of a set X of reals, such that $C_p(X)$ does not have the Pytkeev property. This is another local property of topological spaces: A space X has the *Pytkeev property* if for every $A \subseteq X$ and every $y \in \overline{A} \setminus A$ there is a countable family $\mathcal{A}$ of infinite subsets of A such that every neighbourhood of x contains a member of $\mathcal{A}$.

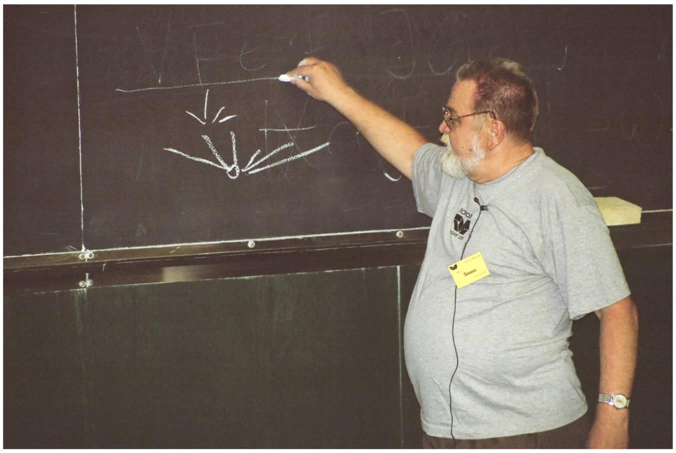
The spaces of continuous functions over the Mrówka-Isbell spaces associated to almost disjoint families in relation to the study of the Lindelöf property in $C_p(X)$ are investigated in [89].

Function spaces with the topology of uniform convergence were considered by Bella and Simon in [42] where it is shown that the set of nowhere constant functions is dense in $C(X, Y)$ if Y is a normed linear space and X is a dense-in-itself normal space or separable completely regular space.

5. Other

In his undergraduate topology classes Petr Simon would maintain that *every respectable topological space is Tychonoff*. Even so, he himself has sinned and occasionally looked at the less respectable ones. We already saw an example of this above [22,31]. In [24], using his *partitionable MAD family* and the so-called *Jones machine* he produced an example of two regular, functionally Hausdorff spaces such that the product of their completely regular modifications does not coincide with the completely regular modification of their product.[7] In a joint paper [46] with Eraldo Giuli he showed that the category of all topological spaces in which every bounded set is Hausdorff is not co-well-powered. In [84], he together with Gino

[7] The *completely regular modification* of a space (X, τ) is X endowed with the weakest topology making all τ-continuous functions continuous.



Petr Simon lecturing on the Čech function during TOPOSYM 2006.

Tironi showed that locally feebly compact first countable regular spaces can be densely embedded into feebly compact first countable regular spaces. In Bela, Costantini and Simon [88] appears a consistent construction of a countably compact Hausdorff space which is Fréchet and contains a copy of the sequential fan.

In a joint paper with Pelant and Vaughan [36] we find information about the minimal number of free prime filters of closed sets on a non-compact space. For completely regular spaces this number is at least $\aleph_2$, while for Hausdorff spaces the best bound they could find is $\aleph_1$.

Simon together with Hindman and van Mill [49] considered $\beta\mathbb{Z}$ as a compact left topological semigroup, and show that there is a strictly increasing chain of principal left ideals and of principal closed ideals.

The paper [91] with Fred Galvin, which answers a 1947 problem of Eduard Čech by constructing a so called *Čech function* — a pathological closure operator on $\mathcal{P}(\omega)$ which is surjective yet not the identity, has a curious history: Galvin knew since 1987 that the existence of a completely separable MAD family suffices, while Balcar, Dočkálková and Simon in [26] (1984) constructed an AD family with similar properties, which would already suffice for the Galvin result. Even though they were both well aware of each others results, it took them 20 more years to put the two facts together.

Convergence structures on groups were treated in [33] and [66]. In the first paper Simon and Fabio Zanolin show that there is a Boolean coarse convergence group that cannot be embedded into a sequentially compact convergence group, while in second one Simon uses the additive group of rational numbers to show that the theory of sequential groups does not admit a reasonable notion of completeness. To exemplify this he shows that the smallest convergence structure on $\mathbb{Q}$ making the sequence $\langle\frac{1}{n}\rangle_n$ converge to 0 is complete, and constructs another group convergence on $\mathbb{Q}$ such that some irrationals, but not all, are limits of Cauchy sequences.

Apart from his research activities, Petr Simon has written several surveys and introductory articles [18,21,40,57,71,93] as well as bibliographical articles dedicated to the life and mathematics of Bohuslav Balcar [96], Eduard Čech [54,55,58,60] (including co-editing the book *The mathematical legacy of Eduard Čech* with Miroslav Katětov [59]), Zdeněk Frolík [43,47,53], Miroslav Katětov [67,70,72] and Jan Pelant [86,87]. This all, of course, reflected his status in the mathematical community in Prague.

6. Questions

The papers co-authored by Petr Simon contain many questions. From our conversations with him we got the impression he would very much have liked to be instrumental in solving the following four.

Question 6.1 *([32]). Must a closed separable subset of $\beta\mathbb{N}$ that is not a retract have a tiny sequence?*

A *tiny sequence* in a space X is a family $\{U_{n,m} : n, m \in \omega\}$ (a matrix) of open sets with the property that for the union $\bigcup_{m\in\omega} U_{n,m}$ of each row is dense, and yet whenever one chooses finite subfamilies $\{U_{n,m} : m \in F_n\}$ of the rows the union $\bigcup_{n\in\omega} \bigcup_{m\in F_n} U_{n,m}$ is not dense. These were introduced by Szymański in [117] and used there in the proof that a particular separable subset of $\beta\mathbb{N}$, constructed from Martin's Axiom, was not a retract. Simon's ZFC-example has a natural tiny sequence, hence the question.

In [114] Leonid Shapiro also constructed a separable non-retract of $\beta\mathbb{N}$. The construction is indirect: after deriving a condition under which the absolute of a compact space is not an absolute retract of $\beta\mathbb{N}$ he constructs a separable compact space of weight $\aleph_1$ that does not meet that condition. We do not know whether this yields in fact a counterexample to this question.

Question 6.2 *([39]).* RPC(ω)*: Given a maximal almost disjoint family $\mathcal{A}$ does there exist an almost disjoint refinement for the family of all $\mathcal{A}$-large sets?*

We discussed this question in Section 3 and all we can say is that it is a beautiful question, both in its combinatorial formulation and its topological equivalents.

Question 6.3 *([44]). Is there (in* ZFC*) a non-meager P-filter?*

We mentioned that Walter Rudin's non-homogeneity proof for $\beta\mathbb{N} \setminus \mathbb{N}$ from [113] used P-points. These points feature, as ultrafilters, in solutions to many combinatorial problems involving the set of natural numbers and quite often they are even necessary for that solution. When Saharon Shelah showed that the existence of P-points in $\beta\mathbb{N} \setminus \mathbb{N}$ is not provable in ZFC alone, see [121], this spurred research into good approximations of P-points whose existence could be shown on basis of ZFC alone. In the paper [44] Petr Simon and his co-authors took up this topic. They focused on the question of how close, in ZFC, a P-filter can be to an ultrafilter and hypothesized that it can be non-meager.[8] That paper contains Petr Simon's proof of the existence of such filters under $\mathfrak{t} = \mathfrak{b}$ or $\mathfrak{b} < \mathfrak{d}$ and shows that their non-existence would need large cardinals. Whether they can be shown to exist in ZFC is still an intriguing open question.

Question 6.4 *([48]). Does every extremally disconnected compact space contain a discretely untouchable point?*

This was also discussed in Section 3. As mentioned there this is an attempt to prove that compact extremally disconnected are not homogeneous by means of an easily formulated property 'shared by some but not all points'. Other properties have been brought to bear on this problem but 'discretely untouchable' seems particularly susceptible to combinatorial treatment.

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[8] As a subset of the Cantor space; since filters here are collections of subsets of the natural numbers, we can think of them as subsets of the Cantor space.

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