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# Joint channel and Doppler estimation for OSDM underwater acoustic communications

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ABSTRACT

Orthogonal signal-division multiplexing (OSDM) has recently emerged as a promising modulation scheme for underwater acoustic communications. Although providing more flexibility in system design, it suffers from a special interference structure over time-varying channels. To enable reliable OSDM equalization, we propose a joint channel impulse response and Doppler estimation algorithm in this paper. Compared to the existing method, which solves a nonconvex bilinear optimization problem iteratively in the frequency domain without a guarantee of global optimality, the proposed algorithm transforms the problem into a convex quadratic minimization in the time domain and can achieve a closed-form solution.



## 1. Introduction

Orthogonal signal-division multiplexing (OSDM) is a generalized modulation scheme which can unify conventional orthogonal frequency-division multiplexing (OFDM) and single-carrier frequency-domain equalization techniques. It was first proposed in [1,2] and shares a similar signal structure with vector OFDM [3,4]. More recently, due to its attractive capability of trading off bandwidth management flexibility with peak-to-average power ratio, OSDM has also been used in underwater acoustic communications [5–8].

A major design challenge for these OSDM communication systems is the severe Doppler effects in underwater acoustic channels, which are typically several orders of magnitude greater than those in terrestrial radio channels [9,10]. Specifically, OSDM divides each block into several vectors. Although over time-invariant channels all these vectors can be decoupled and thus simple per-vector equalization can be used for symbol detection [4,8], in the presence of channel Doppler effects, the orthogonality among these vectors will be destroyed. In this case, inter-vector interference (IVI) will arise, which is analogous to the inter-carrier interference (ICI) in conventional OFDM and may cause significant degradation in OSDM system performance [8].

To enable OSDM equalization in time-varying channels, not only the channel impulse response (CIR) but also the Doppler parameters need to be estimated. In [7], the post-resampling Doppler effect is simply modeled as a carrier frequency offset, and pilot vectors and zero symbols are, respectively, introduced for separate CIR and Doppler estimation. In contrast, an alternating least-squares (ALS) algorithm was recently developed in [8] for joint CIR and Doppler estimation. It can avoid the zero-symbol overhead and is more realistic since it eliminates the single-frequency restriction of the Doppler effect. However, based on a nonconvex bilinear model, the ALS algorithm provides no guarantee of global optimality.

To address this problem, an alternative joint CIR and Doppler estimation algorithm is proposed in this paper. Unlike the ALS algorithm, which estimates IVI iteratively in the frequency domain, the proposed method represents the Doppler effect in the time domain by using a basis expansion model (BEM). As such, Doppler compensation in this case is much easier than the IVI cancellation with the ALS algorithm. More importantly, the joint channel and Doppler estimation is thus reformulated to a convex quadratic optimization problem, which can be directly solved in closed form. Numerical simulations corroborate that a significant performance improvement can be achieved by the proposed algorithm in OSDM underwater acoustic communications.

*Notation*: $(\cdot)^*$ stands for conjugate, $(\cdot)^T$ for transpose, $(\cdot)^H$ for Hermitian transpose, $(\cdot)^\dagger$ for Moore-Penrose pseudo-inverse, and $\|\cdot\|$ for the Euclidean norm. We define $[\mathbf{x}]_n$ as the $n$th entry of the vector $\mathbf{x}$, and $[\mathbf{X}]_{m,n}$ as the $(m,n)$th entry of the matrix $\mathbf{X}$, where

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all indices are starting from 0. Similarly, $[\mathbf{x}]_{m:n}$ indicates the subvector of $\mathbf{x}$ from entry $m$ to $n$, and $[\mathbf{X}]_{m:n,p:q}$ indicates the submatrix of $\mathbf{X}$ from row $m$ to $n$ and from column $p$ to $q$, where only the colon is kept when all rows or columns are included. We use diag$\{\mathbf{x}\}$ to represent a diagonal matrix with $\mathbf{x}$ on its diagonal, and Diag$\{\mathbf{A}_0, \ldots, \mathbf{A}_{N-1}\}$ to represent a block-diagonal matrix created with the submatrices $\{\mathbf{A}_n\}_{n=0}^{N-1}$. Similarly, circ$\{\mathbf{x}\}$ is a circulant matrix whose first column is $\mathbf{x}$, and Circ$\{\mathbf{A}_0, \ldots, \mathbf{A}_{N-1}\}$ is a block-circulant matrix with its first block column being $[\mathbf{A}_0^T, \ldots, \mathbf{A}_{N-1}^T]^T$. Moreover, $\mathbf{I}_N$ refers to the $N \times N$ identity matrix; $\mathbf{F}_N$ stands for the $N \times N$ unitary discrete Fourier transform (DFT) matrix; $\mathbf{0}_N$ ($\mathbf{1}_N$) denotes the $N \times 1$ all-zero (all-one) vector; $\mathbf{i}_N(n)$ and $\mathbf{f}_N(n)$ are the $n$th columns of $\mathbf{I}_N$ and $\mathbf{F}_N$, respectively.

## 2. Problem formulation

Consider a data block $\mathbf{d}$ of $K = MN$ symbols. Unlike conventional OFDM modulation implemented by a single $K$-point inverse DFT (IDFT), OSDM modulation is performed with $M$ IDFTs of length $N$, i.e.,

$$\mathbf{s} = \left(\mathbf{F}_N^H \otimes \mathbf{I}_M\right)\mathbf{d}, \tag{1}$$

where $\mathbf{s}$ is the length-$K$ modulated block and $\otimes$ denotes the Kronecker product. Then, after inserting a cyclic prefix (CP) and upconverting it to the carrier frequency, the OSDM signal is transmitted through an underwater acoustic channel.

At the receiver, by performing front-end resampling, the channel Doppler effect in a wideband underwater acoustic communication signal can be approximately reduced to a narrowband phenomenon [11]. Therefore, the post-resampling Doppler effect is here modeled as a time-varying phase distortion as in [12,13]. Moreover, we assume the channel memory is of length $L$ and denote the CIR as $\mathbf{h} = [h_0, h_1, \ldots, h_L]^T$. As in OFDM, by setting the CP length to $K_g \geq L$, the inter-block interference can be avoided, and accordingly the received OSDM signal after CP removal can be expressed as

$$\mathbf{r} = \tilde{\mathbf{G}}\tilde{\mathbf{H}}\mathbf{s} + \tilde{\mathbf{z}}, \tag{2}$$

where $\tilde{\mathbf{G}} = \text{diag}\{[e^{j\theta_0}, e^{j\theta_1}, \ldots, e^{j\theta_{K-1}}]^T\}$ is the $K \times K$ diagonal phase distortion matrix, $\tilde{\mathbf{H}} = \text{circ}\{[\mathbf{h}^T, \mathbf{0}_{K-L-1}^T]^T\}$ is the $K \times K$ circulant CIR matrix, and $\tilde{\mathbf{z}}$ is the noise term.

Then, the OSDM demodulation at the receiver performs $M$ DFTs of length $N$, thus transforming the received signal back into the frequency domain, i.e.,

$$\mathbf{x} = (\mathbf{F}_N \otimes \mathbf{I}_M)\mathbf{r} = \mathbf{GHd} + \mathbf{z}, \tag{3}$$

where $\mathbf{x}$ and $\mathbf{z}$ are the $K \times 1$ demodulated OSDM signal and noise blocks, respectively; $\mathbf{G} = (\mathbf{F}_N \otimes \mathbf{I}_M)\tilde{\mathbf{G}}(\mathbf{F}_N^H \otimes \mathbf{I}_M)$ and $\mathbf{H} = (\mathbf{F}_N \otimes \mathbf{I}_M)\tilde{\mathbf{H}}(\mathbf{F}_N^H \otimes \mathbf{I}_M)$ are $K \times K$ matrices.

Furthermore, it can be shown that the matrices $\mathbf{G}$ and $\mathbf{H}$ have the following structure [8]:

$$\mathbf{G} = \text{Circ}\{\mathbf{G}_0, \mathbf{G}_1, \ldots, \mathbf{G}_{N-1}\}, \tag{4}$$

$$\mathbf{H} = \text{Diag}\{\mathbf{H}_0, \mathbf{H}_1, \ldots, \mathbf{H}_{N-1}\}, \tag{5}$$

with the $M \times M$ blocks

$$\mathbf{G}_i = \text{diag}\{\mathbf{g}_i\}, \qquad i = 0, 1, \ldots, N-1, \tag{6}$$

where the diagonal entries $[\mathbf{g}_i]_m = \frac{1}{N}\sum_{n=0}^{N-1} e^{j(\theta_{nM+m} - \frac{2\pi}{N}ni)}$ for $m = 0, 1, \ldots, M-1$;

$$\mathbf{H}_n = \mathbf{\Lambda}_M^{nH}\mathbf{F}_M^H\bar{\mathbf{H}}_n\mathbf{F}_M\mathbf{\Lambda}_M^n, \quad n = 0, 1, \ldots, N-1, \tag{7}$$

where $\mathbf{\Lambda}_M^n = \text{diag}\{[1, e^{-j\frac{2\pi n}{K}}, \ldots, e^{-j\frac{2\pi n}{K}(M-1)}]^T\}$, $\bar{\mathbf{H}}_n = \text{diag}\{[H_n, H_{N+n}, \ldots, H_{(M-1)N+n}]^T\}$ and its entries $H_k = \sum_{l=0}^{L} h_l e^{-j\frac{2\pi}{K}lk}$ for $k = 0, 1, \ldots, K-1$.

For the simple case where the channel is time-invariant with $\tilde{\mathbf{G}} = \mathbf{I}_K$, we also have $\mathbf{G} = \mathbf{I}_K$. Then, by partitioning the $K \times 1$ blocks $\mathbf{d}$, $\mathbf{x}$ and $\mathbf{z}$ into $N$ vectors of length $M$, and defining $\mathbf{d}_n = [\mathbf{d}]_{nM:nM+M-1}$, $\mathbf{x}_n = [\mathbf{x}]_{nM:nM+M-1}$ and $\mathbf{z}_n = [\mathbf{z}]_{nM:nM+M-1}$ as the $n$th symbol vector, demodulated vector and noise vector, respectively, it can be readily obtained from (3)–(5) that

$$\mathbf{x}_n = \mathbf{H}_n\mathbf{d}_n + \mathbf{z}_n, \qquad n = 0, \ldots, N-1, \tag{8}$$

which indicates that the detection of the $N$ symbol vectors in the OSDM system can be decoupled. However, in the presence of channel time variations, $\mathbf{G}$ will no longer be a block-diagonal matrix, which leads to IVI among OSDM symbol vectors and thus degrades the system performance. To mitigate the time-varying phase distortion and facilitate the channel equalization that follows, joint channel CIR and Doppler estimation is critical.

## 3. Joint CIR and Doppler estimation

### 3.1. Existing ALS algorithm

We start with a brief introduction of the existing ALS algorithm for joint CIR and Doppler estimation, which is coupled inherently with symbol detection and is performed iteratively in the frequency domain. The corresponding OSDM receiver structure is given in Fig. 1(a).

Specifically, during the initialization step $\gamma = 0$, it is assumed that $\hat{\mathbf{g}}_i^{\langle 0 \rangle} = \delta_i \mathbf{1}_M$ with $\delta_i$ denoting the Kronecker delta, i.e., the channel is assumed to be time-invariant, and a pilot-assisted CIR estimate $\hat{\mathbf{h}}^{\langle 0 \rangle}$ is computed by ignoring IVI. Then, based on (8), we can obtain a tentative OSDM block estimate $\hat{\mathbf{d}}$, by which the CIR and phase distortion can now be jointly estimated by solving

$$\min_{\mathbf{h}, \{\mathbf{g}_i\}} \left\| \mathbf{x} - \mathbf{GH}\hat{\mathbf{d}} \right\|^2. \tag{9}$$

It can be seen that (9) is bilinear in $\mathbf{G}$ and $\mathbf{H}$ and thus the minimization problem is nonconvex. As such, there is no efficient global optimization method for it. To this end, the ALS algorithm tackles the problem via iterative two-step least squares (LS) estimation. Assume that at the $(\gamma - 1)$th iteration we have obtained the estimates $\hat{\mathbf{h}}^{\langle \gamma-1 \rangle}$, $\{\hat{\mathbf{g}}_i^{\langle \gamma-1 \rangle}\}$ and their corresponding matrices $\hat{\mathbf{H}}^{\langle \gamma-1 \rangle}$, $\hat{\mathbf{G}}^{\langle \gamma-1 \rangle}$. Based on these, IVI can be reconstructed and cancelled in the frequency domain to further produce a Doppler-compensated version of $\mathbf{x}$, denoted here by $\hat{\mathbf{x}}^{\langle \gamma-1 \rangle}$. Then, at the $\gamma$th iteration, the first LS step of the algorithm estimates the CIR as

$$\begin{aligned} \hat{\mathbf{h}}^{\langle \gamma \rangle} &= \arg\min_{\mathbf{h}} \left\| \hat{\mathbf{x}}^{\langle \gamma-1 \rangle} - \mathbf{H}\hat{\mathbf{d}} \right\|^2 \\ &= \arg\min_{\mathbf{h}} \left\| \hat{\mathbf{x}}^{\langle \gamma-1 \rangle} - \hat{\mathbf{A}}\mathbf{h} \right\|^2, \end{aligned} \tag{10}$$

and thus $\hat{\mathbf{h}}^{\langle \gamma \rangle} = \hat{\mathbf{A}}^{\dagger}\hat{\mathbf{x}}^{\langle \gamma-1 \rangle}$, where $\hat{\mathbf{A}} = \mathbf{U}^H\hat{\mathbf{D}}\mathbf{P}_{M,N}\bar{\mathbf{F}}_K$, with $\mathbf{U} = \text{Diag}\{\mathbf{F}_M\mathbf{\Lambda}_M^0, \mathbf{F}_M\mathbf{\Lambda}_M^1, \ldots, \mathbf{F}_M\mathbf{\Lambda}_M^{N-1}\}$, $\hat{\mathbf{D}} = \text{diag}\{\mathbf{U}\hat{\mathbf{d}}\}$, $\bar{\mathbf{F}}_K = \sqrt{K}[\mathbf{F}_K]_{:,0:L}$, and

$$\mathbf{P}_{M,N} = \begin{bmatrix} \mathbf{I}_M \otimes \mathbf{i}_N^T(0) \\ \mathbf{I}_M \otimes \mathbf{i}_N^T(1) \\ \vdots \\ \mathbf{I}_M \otimes \mathbf{i}_N^T(N-1) \end{bmatrix}.$$

Subsequently, based on the CIR updated in (10), the second LS step estimates the phase distortion as

$$\begin{aligned} \hat{\mathbf{g}}^{\langle \gamma \rangle} &= \arg\min_{\mathbf{g}} \left\| \mathbf{x} - \mathbf{G}\hat{\mathbf{H}}^{\langle \gamma \rangle}\hat{\mathbf{d}} \right\|^2 \\ &= \arg\min_{\mathbf{g}} \left\| \mathbf{x} - \hat{\mathbf{B}}^{\langle \gamma \rangle}\mathbf{g} \right\|^2, \end{aligned} \tag{11}$$

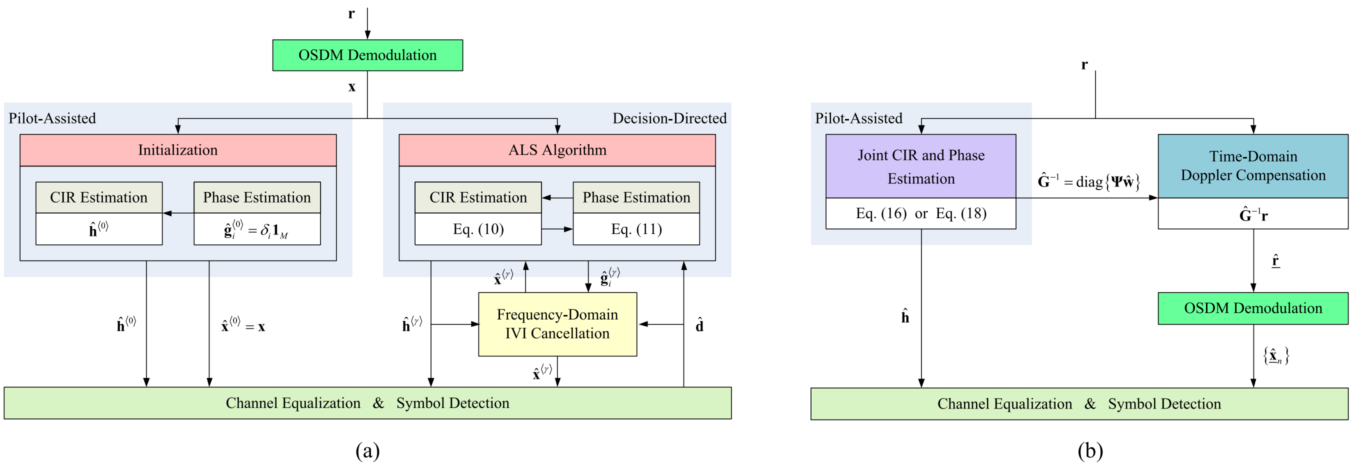


**Fig. 1.** Block diagram of OSDM receivers. (a) the ALS-based receiver; (b) the receiver using the proposed joint CIR and Doppler estimation algorithm.

or equivalently, $\hat{\mathbf{g}}^{\langle\gamma\rangle} = \hat{\mathbf{B}}^{\langle\gamma\rangle\dagger}\mathbf{x}$. Here, we can exploit the slow variation property of the post-resampling phase distortion and make the approximation that $\mathbf{g}_i = \mathbf{0}_M$ for $I < i < N - I$ [8]. Thus, in (11), we have $\mathbf{g} = [\mathbf{g}_{-I}^T, \dots, \mathbf{g}_I^T]^T$ and $\hat{\mathbf{B}}^{\langle\gamma\rangle}$ is a $K \times (2I+1)M$ matrix with its $(n, i)$th block

$$[\hat{\mathbf{B}}^{\langle\gamma\rangle}]_{nM:nM+M-1,iM:iM+M-1} = \text{diag}\{\hat{\mathbf{H}}^{\langle\gamma\rangle}_{n-i+I}\hat{\mathbf{d}}_{n-i+I}\}$$

for $n = 0, 1, \dots, N-1$ and $i = 0, 1, \dots, 2I$, where all subscripts are taken modulo-$N$.

The iterations continue until $\gamma$ reaches a given number $\gamma_{\max}$, then $\hat{\mathbf{d}}$ is updated and the ALS algorithm repeats [8]. This coupling process of channel estimation and symbol detection terminates when a specified number of rounds have elapsed or there is no significant decline in the objective function in (9).

### 3.2. Proposed algorithm

The ALS-based channel estimation has a major drawback. It can only find a local optimum in the neighborhood of the initial point, i.e., the time-invariant channel estimates $\hat{\mathbf{h}}^{\langle 0\rangle}$ and $\{\hat{\mathbf{g}}_i^{\langle 0\rangle}\}$; due to the nonconvexity of (9), there is no guarantee that the global optimal solution can be achieved. To this end, an alternative joint CIR and Doppler estimation algorithm is proposed in this paper. Instead of performing frequency-domain IVI cancellation as in the ALS algorithm, here time-domain phase distortion compensation is adopted based on (2), which can be easier to implement. Specifically, by defining $\mathbf{q} = [e^{-j\theta_0}, e^{-j\theta_1}, \dots, e^{-j\theta_{K-1}}]^T$, we have $\tilde{\mathbf{G}}^{-1} = \text{diag}\{\mathbf{q}\}$, and the Doppler-compensated received signal is $\underline{\mathbf{r}} = \tilde{\mathbf{G}}^{-1}\mathbf{r}$. In this case, the $n$th demodulated vector can be rewritten as

$$\underline{\mathbf{x}}_n = \left[\mathbf{f}_N^T(n) \otimes \mathbf{I}_M\right]\underline{\mathbf{r}} = \boldsymbol{\Omega}_n\mathbf{q}, \tag{12}$$

where $\boldsymbol{\Omega}_n = [\mathbf{f}_N^T(n) \otimes \mathbf{I}_M]\text{diag}\{\mathbf{r}\}$ is an $M \times K$ matrix. And according to (7) and (8), it can also be derived that

$$\underline{\mathbf{x}}_n = \mathbf{H}_n\mathbf{d}_n + \underline{\mathbf{z}}_n = \mathbf{A}_n\mathbf{h} + \underline{\mathbf{z}}_n, \tag{13}$$

where $\mathbf{A}_n = \mathbf{U}_n^H\mathbf{D}_n\boldsymbol{\Gamma}_n$ is an $M \times (L+1)$ matrix with $\mathbf{U}_n = \mathbf{F}_M\boldsymbol{\Lambda}_M^n$, $\mathbf{D}_n = \text{diag}\{\mathbf{U}_n\mathbf{d}_n\}$ and $\boldsymbol{\Gamma}_n = [\mathbf{I}_M \otimes \mathbf{i}_N^T(n)]\bar{\mathbf{F}}_K$; $\underline{\mathbf{z}}_n$ is the demodulated noise term.

Assume that there are $P$ pilot vectors in each block and their index set is $S_{\mathcal{P}} = \{n_0, n_1, \dots, n_{P-1}\}$, i.e., $\{\mathbf{d}_n | n \in S_{\mathcal{P}}\}$ are known to the OSDM receiver. By stacking all pilot vectors together, based on (12) and (13), we have

$$\boldsymbol{\Omega}_{\mathcal{P}}\mathbf{q} = \mathbf{A}_{\mathcal{P}}\mathbf{h} + \underline{\mathbf{z}}_{\mathcal{P}}, \tag{14}$$

where $\boldsymbol{\Omega}_{\mathcal{P}} = [\boldsymbol{\Omega}_{n_0}^T, \dots, \boldsymbol{\Omega}_{n_{P-1}}^T]^T$, $\mathbf{A}_{\mathcal{P}} = [\mathbf{A}_{n_0}^T, \dots, \mathbf{A}_{n_{P-1}}^T]^T$ and $\underline{\mathbf{z}}_{\mathcal{P}} = [\underline{\mathbf{z}}_{n_0}^T, \dots, \underline{\mathbf{z}}_{n_{P-1}}^T]^T$. Since there are a total of $K+L+1$ unknowns in $\mathbf{q}$ and $\mathbf{h}$, while only $MP < K$ linear equations are available in (14), the system here is actually underdetermined.

To reduce the unknowns in (14), we further introduce a BEM [14] to represent the phase distortion, i.e.,

$$\mathbf{q} = \boldsymbol{\Psi}\mathbf{w}, \tag{15}$$

where $\boldsymbol{\Psi}$ is the $K \times J$ matrix with $J$ basis vectors as its columns, and $\mathbf{w}$ is the corresponding $J \times 1$ BEM coefficient vector. Then, by judiciously selecting the basis vectors, it is possible to make $J \ll K$. For instance, we can invoke the slow variation property of the phase distortion as in the ALS algorithm, and thus use a set of $J = 2Q+1$ complex exponential basis vectors $\mathbf{f}_K(K-Q), \dots, \mathbf{f}_K(K-1), \mathbf{f}_K(0), \dots, \mathbf{f}_K(Q)$. In this case, assuming the maximum Doppler spread is $f_{\max}$ and the symbol period is $T_s$, we have $Q = \lceil f_{\max}KT_s \rceil$, where $\lceil \cdot \rceil$ denotes the integer ceiling. As a result, the phase distortion can be efficiently parameterized with much fewer basis coefficients, and it will not be difficult to have more equations than the number of unknowns in (14), i.e., $MP \geq (J+L+1)$.

Moreover, without loss of generality[1], we can assume $\theta_0 = 0$ and thus $\mathbf{i}_K^T(0)\mathbf{q} = 1$. Based on (14) and (15), the proposed joint channel and Doppler estimation algorithm is to solve

$$\min_{\mathbf{h},\mathbf{w}} \quad \|\boldsymbol{\Omega}_{\mathcal{P}}\boldsymbol{\Psi}\mathbf{w} - \mathbf{A}_{\mathcal{P}}\mathbf{h}\|^2$$
$$\text{s.t.} \quad \mathbf{i}_K^T(0)\boldsymbol{\Psi}\mathbf{w} = 1, \tag{16}$$

where the constraint also avoids the pathological point $\mathbf{h} = \mathbf{0}_{L+1}$ and $\mathbf{w} = \mathbf{0}_J$. Once the channel estimates $\hat{\mathbf{h}}$ and $\hat{\mathbf{w}}$ are computed, the time-domain Doppler compensation is performed by $\hat{\underline{\mathbf{r}}} = \hat{\mathbf{G}}^{-1}\mathbf{r}$, where $\hat{\mathbf{G}}^{-1} = \text{diag}\left\{\boldsymbol{\Psi}\hat{\mathbf{w}}\right\}$. Then, OSDM demodulation follows and the resulting vectors $\{\hat{\underline{\mathbf{x}}}_n\}$ are fed to channel equalization and symbol detection. The corresponding receiver structure is shown in Fig. 1(b).

Compared to the bilinear problem in (9), here (16) is a quadratic problem with an affine constraint, which is equivalent to finding the minimum-norm solution of a linear equation. Therefore, it is convex and global optimality can be guaranteed. Specifically, by defining

$$\mathbf{c} = \begin{bmatrix} \mathbf{h} \\ \mathbf{w} \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} \mathbf{0}_{L+1} \\ \boldsymbol{\Psi}^T\mathbf{i}_K(0) \end{bmatrix}, \quad \boldsymbol{\Pi}_{\mathcal{P}} = [-\mathbf{A}_{\mathcal{P}}, \boldsymbol{\Omega}_{\mathcal{P}}\boldsymbol{\Psi}],$$

[1] Any initial phase distortion $\theta_0$ scales the corresponding CIR estimate by $e^{-j\theta_0}$. The channel matrix multiplication $\tilde{\mathbf{G}}\tilde{\mathbf{H}}$ in (2) remains constant and thus the value of $\theta_0$ has no effect on the final symbol detection.

and $\mathbf{R}_{\mathcal{P}} = \mathbf{\Pi}_{\mathcal{P}}^H \mathbf{\Pi}_{\mathcal{P}}$, we can even obtain the closed-form solution of the joint channel estimation problem in (16) as

$$\hat{\mathbf{c}} = \mathbf{R}_{\mathcal{P}}^{-1}\mathbf{a}/(\mathbf{a}^H \mathbf{R}_{\mathcal{P}}^{-1}\mathbf{a}). \tag{17}$$

**Remark 1** (Uniqueness of the channel estimate)**.** It should be noted that in (17), we have actually assumed $\mathbf{\Pi}_{\mathcal{P}}$ has full column rank; otherwise, the optimization problem in (16) may have multiple (or even infinite) solutions in the nullspace of $\mathbf{R}_{\mathcal{P}}$. To guarantee the uniqueness of the joint channel and Doppler estimate $\hat{\mathbf{c}}$, it is required that $\mathbf{R}_{\mathcal{P}}$ is positive definite. To this end, a diagonal loading term can be added, i.e.,

$$\begin{aligned} \min_{\mathbf{h},\mathbf{w}} \quad & \|\mathbf{\Omega}_{\mathcal{P}}\mathbf{\Psi}\mathbf{w} - \mathbf{A}_{\mathcal{P}}\mathbf{h}\|^2 + \lambda\left(\|\mathbf{h}\|^2 + \|\mathbf{w}\|^2\right) \\ \text{s.t.} \quad & \mathbf{i}_K^T(0)\mathbf{\Psi}\mathbf{w} = 1, \end{aligned} \tag{18}$$

where $\lambda > 0$ is the diagonal loading parameter. In this case, the estimate has a similar form as that in (17) with only $\mathbf{R}_{\mathcal{P}}$ replaced by $\mathbf{R}'_{\mathcal{P}} = \mathbf{\Pi}_{\mathcal{P}}^H \mathbf{\Pi}_{\mathcal{P}} + \lambda \mathbf{I}_{J+L+1}$.

**Remark 2** (Coupling with symbol detection)**.** Since only the $P$ pilot vectors $\{\mathbf{d}_n | n \in S_{\mathcal{P}}\}$ are involved in (17), the proposed algorithm is actually a simple one-shot estimator. However, it is also straightforward to couple this channel estimation with symbol detection as in the ALS algorithm. In this case, (17) provides only the initial CIR and Doppler estimation. After that, the following rounds operate in a decision-feedback mode, which invokes an optimization problem different from (16), i.e.,

$$\begin{aligned} \min_{\mathbf{h},\mathbf{w}} \quad & \left\|\hat{\mathbf{\Omega}}\mathbf{\Psi}\mathbf{w} - \hat{\mathbf{A}}\mathbf{h}\right\|^2 \\ \text{s.t.} \quad & \mathbf{i}_K^T(0)\mathbf{\Psi}\mathbf{w} = 1, \end{aligned} \tag{19}$$

where $\hat{\mathbf{\Omega}}$ and $\hat{\mathbf{A}}$ are based on the entire OSDM block estimate $\hat{\mathbf{d}}$ obtained from the previous round. Its closed-form solution can be likewise computed as (17) by replacing $\mathbf{R}_{\mathcal{P}}$ with $\hat{\mathbf{R}} = \hat{\mathbf{\Pi}}^H\hat{\mathbf{\Pi}}$, where $\hat{\mathbf{\Pi}} = [-\hat{\mathbf{A}}, \hat{\mathbf{\Omega}}\mathbf{\Psi}]$. As will be shown in Section 4, when the block estimate $\hat{\mathbf{d}}$ is good enough, this symbol-detection-coupled method can usually offer an improved performance compared to the one-shot estimator.

## 4. Simulation results

Consider an OSDM underwater acoustic communication system with $K = 1024$, $M = 16$ and $N = 64$. Each OSDM block is constituted by uncoded QPSK symbols of duration $T_s = 0.125$ ms and a carrier frequency $f_c = 14$ kHz is adopted. The simulated channel consists of 6 discrete paths, whose amplitudes are assumed to be Gaussian distributed (i.e. Rayleigh fading) and delays are uniformly distributed with a maximum spread equal to $\tau_{\max} = 8$ ms, which corresponds to $L = \tau_{\max}/T_s = 64$. Meanwhile, the channel Doppler effect is simulated by a post-resampling Doppler scaling factor $\Delta$.

At the receiver, the complex exponential BEM is used and we set $Q = 5$. Also, $P = 8$ pilot vectors are placed in each block to satisfy the condition $MP > J + L + 1$. In Fig. 2, the bit error rate (BER) performance of the OSDM receivers is evaluated versus signal-to-noise ratio (SNR). We consider the proposed algorithm with one-shot estimation and the ALS algorithm with 0 and 3 iterations. The Doppler scaling factor is here fixed to $\Delta = 4 \times 10^{-5}$ and the system performance over the time-invariant channel ($\Delta = 0$) is also included as a benchmark. It can be seen that there is a performance crossover between the proposed and the ALS algorithms. At low SNRs, the proposed algorithm causes a larger BER compared to the ALS algorithm. This is because the channel noise dominates the Doppler interference in this case; therefore, (16) actually performs noise fitting instead of Doppler estimation. However, as the SNR increases, the IVI becomes more pronounced and the proposed algorithm starts to outperform the ALS algorithm.

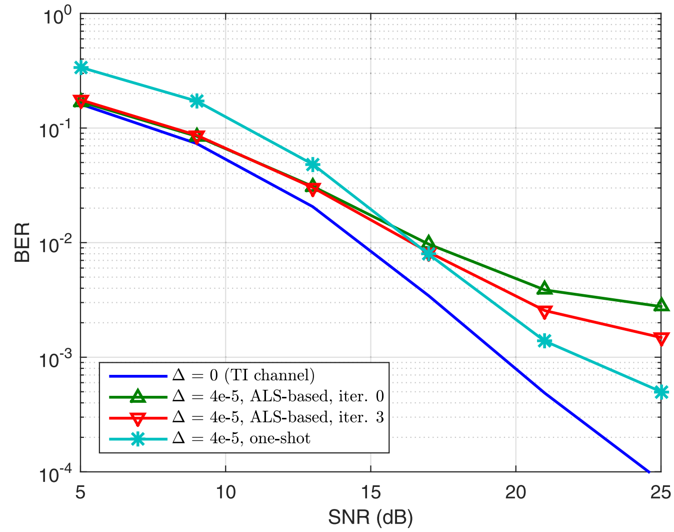


**Fig. 2.** BER performance of the OSDM receivers versus SNR at Doppler scaling factor $\Delta = 4 \times 10^{-5}$.

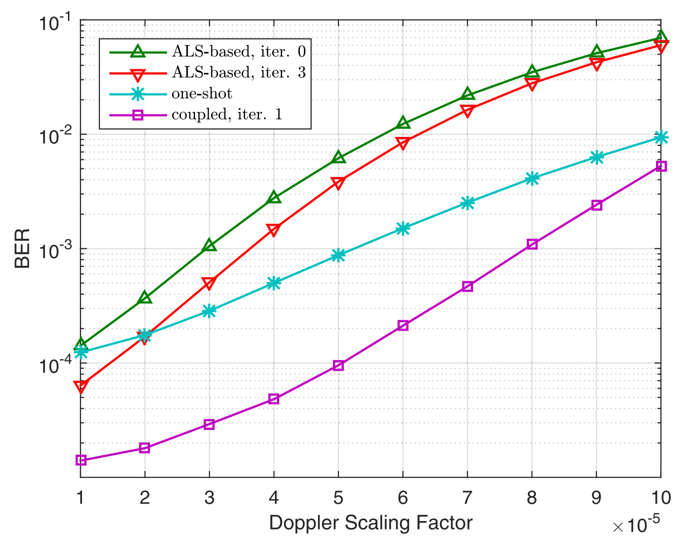


**Fig. 3.** BER performance of the OSDM receivers versus Doppler scaling factor at SNR = 25 dB.

Furthermore, in Fig. 3, the SNR is fixed to 25 dB and the Doppler scaling factor $\Delta \in [0.1, 1] \times 10^{-4}$. We here evaluate also the performance of the proposed algorithm with symbol-detection-coupled estimation in Remark 2. It can be seen that, when $\Delta$ is small, the ALS iterations can effectively improve the system performance, and with three iterations the resulting BER is lower than that of the proposed one-shot estimator for $\Delta \leq 2 \times 10^{-5}$. The observation should not be surprising since there is only modest IVI present in this case. As such, the ALS algorithm is sufficient to produce a reliable block estimate $\hat{\mathbf{d}}$ at the initial iteration, based on which its following iterations take nearly the true block as prior knowledge. Therefore, compared to the proposed algorithm with one-shot estimation, which uses only $P$ pilot vectors, the ALS algorithm can achieve a better performance at small $\Delta$'s. However, when the proposed algorithm is similarly coupled with symbol detection to exploit also the entire block estimate $\hat{\mathbf{d}}$, even with one more iteration, its performance advantage is restored. Moreover, as $\Delta$ increases, the performance gain obtained by the ALS iterations gets reduced, since the initial time-invariant channel estimate may be far from the optimal point and (9) converges to a local minimum. In contrast, the proposed algorithm shows a much lower BER when the IVI becomes the dominant interference.

## 5. Conclusion

A joint CIR and Doppler estimation algorithm is proposed for OSDM underwater acoustic communications. Compared to the existing ALS algorithm, it has three attractive features: (1) Global optimality can be guaranteed thanks to the convex quadratic formulation based on BEM approximations of the Doppler effect. (2) There is always a closed-form solution for one-shot and symbol-detection-coupled estimation. (3) The Doppler distortion is estimated in the time domain and thus more convenient to compensate. Numerical simulations suggest that the proposed algorithm can make the OSDM system more reliable for time-varying channels.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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